《应用随机过程》勘误表

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P.57, line -9, $P_i(\epsilon_A < \sigma_i) = 1$ 应改为 $P_i(\epsilon_A < \infty) = 1$.
P.17, line -11 练习题 1 需要增加条件 $i \neq j$.
P.18, 练习题 12 宜移至 §13, 此题需要用到定理 1.6.3 以及 Wald 等式.
P.3. lines 14 & 15 字符 A!" 是多余的, 应当删除.
P.67, 命题 2.1.3 的证明中有一个符号上的错误. 其证明连同命题 2.1.2 的证明可以修改如次.

命题 2.1.2 的证明:

$$P(X_t = k) = P(S_k \le t < S_{k+1}) = P(\xi_1 + \cdots + \xi_k \le t < \xi_1 + \cdots + \xi_k + \xi_{k+1})$$
$$= \int \cdots \int_D \prod_{j=1}^{k+1} (\lambda e^{-\lambda x_j}) dx_1 \cdots dx_k dx_{k+1}$$

其中积分区域 $D = \{(x_1, x_2, \cdots, x_k, x_{k+1}); x_1 + \cdots + x_k \le t < x_1 + \cdots + x_k + x_{k+1}, x_j \ge 0\}$.
若记 $s_k = x_1 + \cdots + x_k$, $C(k,t) = \{(x_1, x_2, \cdots, x_k); s_k \le t; , x_j \ge 0, 1 \le j \le k\}$. 则

$$\begin{aligned} P(X_t = k) &= \int \cdots \int_{C(k,t)} \left( \int_{t-s_k x_j}^{\infty} \lambda e^{-\lambda x_{k+1}} dx_{k+1} \right) \lambda^k e^{-\lambda s_k} dx_1 \cdots dx_k \\ &= \int \cdots \int_{C(k,t)} e^{-\lambda(t-s_k)} \lambda^k e^{-\lambda s_k} dx_1 \cdots dx_k \\ &= \lambda^k e^{-\lambda t} \int \cdots \int_{C(k,t)} dx_1 \cdots dx_k = \lambda^k e^{-\lambda t} t^k / k! \end{aligned}$$

命题 2.1.3 的证明:

$$P(X_t = k, X_{t+s} - X_t = l) = P(X_t = k, X_{t+s} = k + l)$$
$$= P(S_k \le t < S_{k+1}, S_{k+l} \le t + s < S_{k+l+1})$$
$$= P(\xi_1 + \cdots + \xi_k \le t < \xi_1 + \cdots + \xi_k + \xi_{k+1}; \xi_1 + \cdots + \xi_{k+l} \le t + s < \xi_1 + \cdots + \xi_k + \xi_{k+l+1})$$
$$= \int \cdots \int_F \prod_{j=1}^{k+l+1} (\lambda e^{-\lambda x_j}) dx_1 \cdots dx_k \cdots dx_{k+l+1}$$

其中积分区域,

$$F = \left\{(x_1, x_2, \cdots, x_{k+l+1}); \begin{array}{c} x_1 + \cdots + x_k \le t < x_1 + \cdots + x_{k+1} \\ x_1 + \cdots + x_{k+l} \le t + s < x_1 + \cdots + x_{k+l+1} \\ x_j \ge 0, 1 \le j \le k + l + 1 \end{array} \right\}.$$

若记 $s_k = x_1 + \cdots + x_k$, $r = x_1 + \cdots + x_{k+l}$,

$$G = \{(x_1, x_2, \cdots, x_{k+l}); s_k \le t < s_k + x_{k+1}, r \le t + s\}.$$

则

$$\begin{aligned} P(X_t = k, X_{t+s} = k + l) &= \int \cdots \int_G \left( \int_{t+s-r}^{\infty} \lambda e^{-\lambda x_{k+l+1}} dx_{k+l+1} \right) \lambda^{k+l} e^{-\lambda r} dx_1 \cdots dx_{k+l} \\ &= \int \cdots \int_G e^{-\lambda(t+s-r)} \lambda^k e^{-\lambda r} dx_1 \cdots dx_{k+l} \\ &= \lambda^k e^{-\lambda(t+s)} \int \cdots \int_G dx_1 \cdots dx_{k+l} \end{aligned}$$

做变量变换，令 $y_{k+1} = x_{k+1} - (t - s_k)$, 对于其他 $j \neq k+1$, $y_j = x_j$, 则此变换的 Jacobian 为 1 , 区域 $G$ 经此变换变为

$$G' = \{(y_1, y_2, \cdots, y_{k+l}); y_j \geq 0, y_1 + \cdots y_k \leq t, y_{k+1} + \cdots + y_{k+l} < s\}.$$

$$\begin{gathered} P(X_t = k, X_{t+s} = k+l) = \lambda^k e^{-\lambda(t+s)} \int \cdots \int_{G'} dy_1 \cdots dy_{k+l} \\ = \lambda^k e^{-\lambda(t+s)} (\int \cdots \int_{C(k,t)} dy_1 \cdots dy_k)(\int \cdots \int_{C(l,s)} dy_{k+1} \cdots dy_{k+l}) \\ = \lambda^k e^{-\lambda(t+s)} \frac{t^k}{k!} \frac{s^l}{l!} = P(X_t = k) P(X_s = l) \end{gathered}$$

这个证明的不好之处在于掩盖了概率含义. 在做变量变换时我们其实利用了指数分布的无记忆性.

$$P(\xi > s + t | \xi > s) = P(\xi > t).$$